Section 6: #1-17 EOO, 20, #21-25 odds
#27-39 EOO

(1) $\frac{1}{2}+\frac{4}{4}+\frac{9}{8}+\frac{16}{16}+\frac{25}{32}+\frac{36}{64}\ldots\frac{n^2}{2^n}$

$$\sum_{n=1}^{\infty}\frac{n^2}{2^n} \qquad a_n=\frac{n^2}{2^n} \qquad a_{n+1}=\frac{(n+1)^2}{2^{n+1}}$$

$$\lim_{n\to\infty}\frac{(n+1)^2}{2^{n+1}}\cdot\frac{2^n}{n^2}=$$

$$\lim_{n\to\infty}\frac{(n+1)^2}{2n^2}=\frac{1}{2}<1$$

* therefore the series (Converges) by the Ratio Test.

(5) $$\sum_{n=0}^{\infty}\frac{n!}{(2n)!} \qquad a_n=\frac{n!}{(2n)!} \qquad a_{n+1}=\frac{(n+1)!}{(2(n+1))!}$$

$$\lim_{n\to\infty}\frac{(n+1)!}{(2n+2)!}\cdot\frac{(2n)!}{n!}=$$

$$\lim_{n\to\infty}\frac{(n+1)\cdot(n)\cdot(n-1)\cdot(n-2)\ldots1\cdot(2n)\cdot(2n-1)(2n-2)\ldots1}{(2n+2)\cdot(2n+1)\cdot(2n)\cdot(2n-1)\ldots1\cdot(n)(n-1)(n-2)\ldots1}$$

$$\lim_{n\to\infty}\frac{(n+1)}{2(n+1)\cdot(2n+1)}=\lim_{n\to\infty}\frac{1}{2(2n+1)}=0<1$$

* therefore the series (Converges) by the Ratio Test.

⑨ $\sum_{n=1}^{\infty} n\left(\frac{4}{5}\right)^n$ $a_n = n\left(\frac{4}{5}\right)^n$

$a_{n+1} = (n+1)\left(\frac{4}{5}\right)^{n+1}$

$\lim_{n\to\infty} \frac{(n+1)}{1}\left(\frac{4^{n+1}}{5^{n+1}}\right)\cdot\left(\frac{5^n}{4^n}\right)\cdot\frac{1}{n} =$

$\lim_{n\to\infty} \frac{(n+1)}{n}\frac{4^n\cdot 4\cdot 5^n}{5^n\cdot 5\cdot 4^n} = \lim_{n\to\infty}\frac{4(n+1)}{5(n)} = \frac{4}{5} < 1$ ✓

* Therefore series (converges) by the ratio test.

⑬ $\frac{2}{6}+\frac{3}{7}+\frac{4}{8}+\frac{5}{9}+\ldots$ $\lim_{n\to\infty}\frac{n+1}{n+5} = ①$

$\sum_{n=1}^{\infty}\frac{n+1}{n+5} \Rightarrow$ (Diverges by nth term test!) ✓

⑰ $\sum_{n=1}^{\infty}\frac{n!}{2n^5} =$ $a_n = \frac{n!}{2n^5}$ $a_{n+1} = \frac{(n+1)!}{2(n+1)^5}$

$\lim_{n\to\infty}\frac{(n+1)!}{2(n+1)^5}\cdot\frac{2n^5}{n!} = \lim_{n\to\infty}\frac{(n+1)\cdot n^5}{(n+1)^5} = \infty$

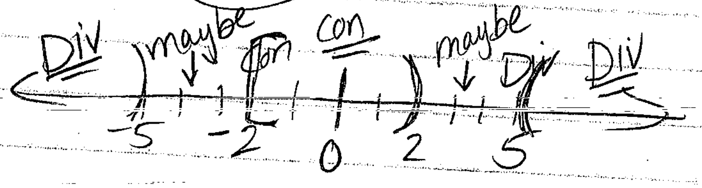
$\infty > 1$ ✓

* So Series (Diverges) by the Ratio Test

⑳ ⓐ Yes, center $x=-2$
ⓑ No, center keeps changing
ⓒ Yes, center $x=3$
ⓓ No, No center $x^2+4\neq 0$
ⓔ Yes, center $x=-1$
ⓕ No, $\tan^n x$ is not of the form $(x-a)^n$.

(21) $\sum_{n=0}^{\infty} c_n x^n$ converges @ $x=-2$
diverges @ $x=5$

Center ($x=0$)

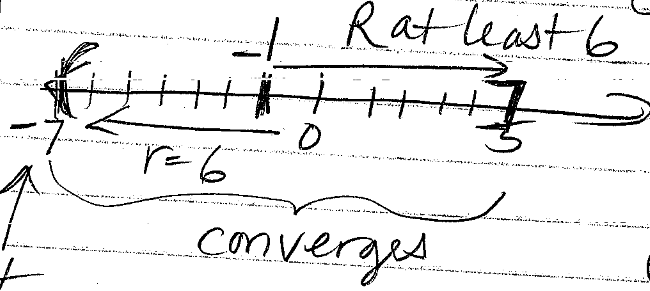


Radius is at least 2 & at most 5

(a) $r=2$ (b) $r=5$ (c) converges: $-1, 0, 1,$
diverges: $-8,$
maybe: $2, 4, -5$

Can't be sure what happens at endpoints either → so maybe on 2 & −5.

(23) $\sum_{n=0}^{\infty} c_n (x+1)^n$ converges at $x=5$
Center $x=-1$



* Radius is at least 6

Not Sure about endpoints of interval

(a) No, Not Symmetric (b) No, Not Symmetric
(c) No, too big (d) Yes $[-7, 5]$

(25) $\sum_{n=0}^{\infty} f_n(x)$

* Converges at $x=5$ & $x=8$ but not at $x=6$

No → power series converge on a continuous interval.

(27) $\frac{(x-2)}{1\cdot 2} + \frac{(x-2)^2}{2\cdot 4} + \frac{(x-2)^3}{3\cdot 8} + \frac{(x-2)^4}{4\cdot 6} + \ldots$

$\sum_{n=1}^{\infty} \frac{(x-2)^n}{n\cdot 2^n} \implies$ use Ratio Test!

$a_n = \frac{(x-2)^n}{n\cdot 2^n}$ $a_{n+1} = \frac{(x-2)^{n+1}}{(n+1)2^{n+1}}$

$\lim_{n\to\infty} \frac{(x-2)^{n+1}}{(n+1)2^{n+1}} \cdot \frac{n\cdot 2^n}{(x-2)^n} =$

$\lim_{n\to\infty} \frac{(x-2)^1 \cdot n}{2\ (n+1)} = \left|\frac{x-2}{2}\right| < 1$ to Converge

$-1 < \frac{x-2}{2} < 1$

$-2 < x-2 < 2$
$+2 \quad\ +2 \quad +2$

$0 < x < 4$
$R = 2$ ✓
$C = 2$

(31) $\sum_{n=1}^{\infty} \frac{n!\,x^n}{n^3 4^n}$ $a_n = \frac{n!\,x^n}{n^3 4^n}$ $a_{n+1} = \frac{(n+1)!\,x^{n+1}}{(n+1)^3 4^{n+1}}$

$\lim_{n\to\infty} \frac{(n+1)!\,x^{n+1}}{(n+1)^3 4^{n+1}} \cdot \frac{n^3 \cdot 4^n}{n!\,x^n} =$ *Use Ratio Test

$\lim_{n\to\infty} \frac{(n+1)\cdot x \cdot n^3}{(n+1)^3 \cdot 4} = \lim_{n\to\infty} \frac{x n^3}{4(n+1)^2} = \infty > 1$

*Therefore the series Diverges for All x-values.

So center: $x=0$ and $R=0$ ✓

(35) $\sum_{n=1}^{\infty} (-1)^n \cdot \frac{x^n}{n^2}$ $a_n = (-1)^n \frac{x^n}{n^2}$ $a_{n+1} = (-1)^{n+1} \cdot \frac{x^{n+1}}{(n+1)^2}$

Ratio Test

$\lim_{n\to\infty} \frac{(-1)^{n+1} x^{n+1}}{(n+1)^2} \cdot \frac{n^2}{(-1)^n \cdot x^n} =$

$\lim_{n\to\infty} \frac{(-1)^1 x \cdot n^2}{(n+1)^2} = |-x| < 1$ to converge

so $-1 < x < 1$ & $(R = 1)$ ✓
center $x = 0$

(39) $\sum_{n=0}^{\infty} (-1)^n \cdot \frac{x^{2n+1}}{(2n+1)!}$ $a_n = \frac{(-1)^n \cdot x^{2n+1}}{(2n+1)!}$

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} =$ $a_{n+1} = \frac{(-1)^{n+1} x^{2n+2+1}}{(2n+2+1)!}$

$\lim_{n\to\infty} \frac{(-1)^{n+1} x^{2n+3}}{(2n+3)!} \cdot \frac{(2n+1)!}{(-1)^n x^{2n+1}} =$

$\lim_{n\to\infty} \frac{(-1) \cdot x^3 \cdot}{(2n+3)(2n+2) \cdot x} = \lim_{n\to\infty} \frac{(-1)x^2}{(2n+3)(2n+2)} = (0)$

Since $0 < 1$, * Always! the series CONVERGES for all x-values.

(Therefore, $R = \infty$) ✓